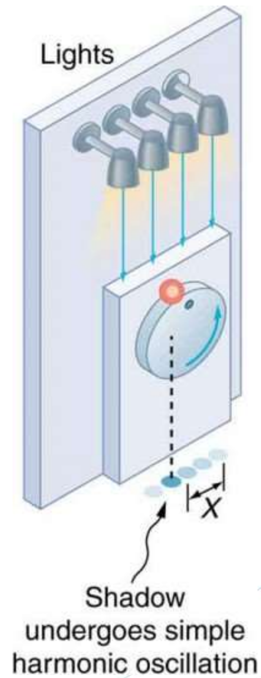
Uniform Circular Motion and Simple Harmonic Motion

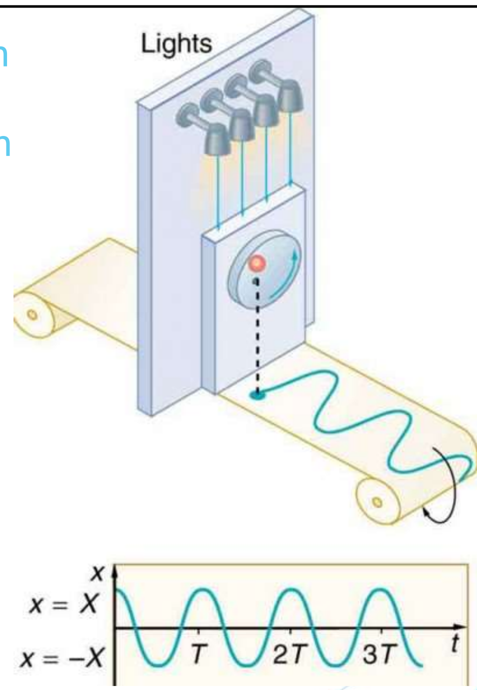
- Simple Harmonic Motion can be described as *the projection* of uniform circular motion.



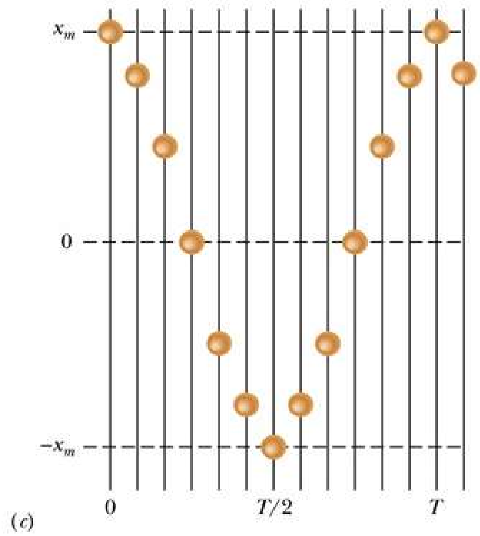
Uniform Circular Motion and Simple Harmonic Motion

- If we add a second dimension, we get our familiar cos function!
- They are all related! Hurray!
- The angular frequency (ω) is:

$$\omega = \frac{2\pi}{T} = 2\pi f$$



Our Old Friend the Mass on a Spring System



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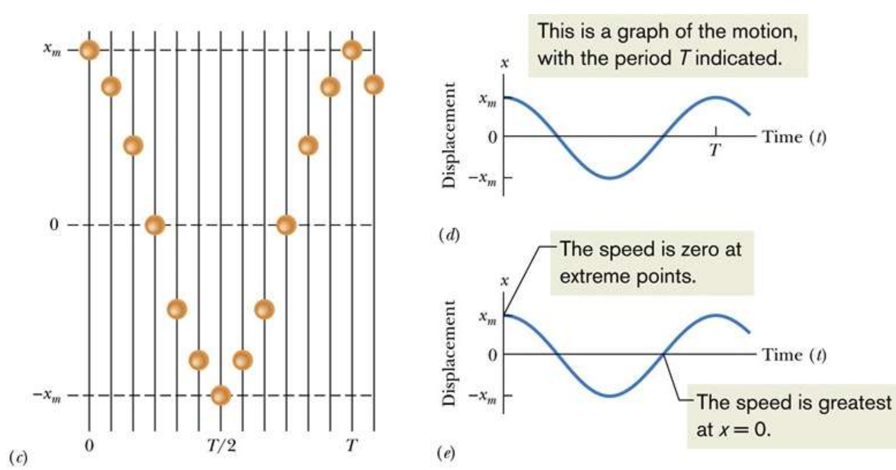
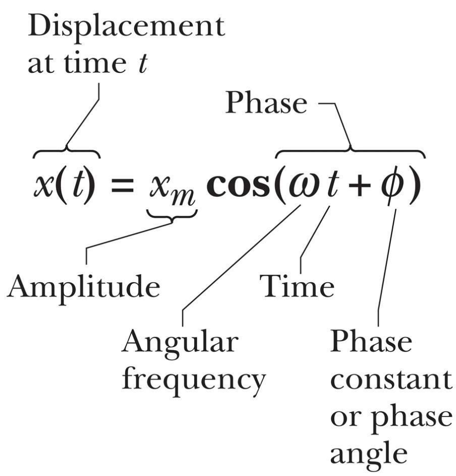


What's the general equation for that trig function?

$$x(t) = x_m \cos(\omega t + \phi)$$

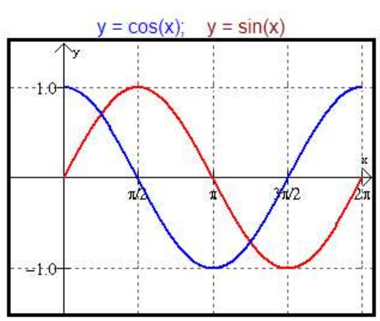


Phase Constant, Phase Angle, & Phase Shift

The graphs of sine and cosine are the same when sine is shifted left by 90° or $\frac{\pi}{2}$ radians. Such a shifting is referred to as a **horizontal shift**.

$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$ shift sine to the left to create cosine

$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ shift cosine to the right to create sine



For the most part, we'll just use:

$$x(t) = x_m \cos(\omega t)$$

In other words, the position of the Simple Harmonic Motion is a function of the amplitude and the angular velocity (or frequency).

That's position/displacement, what about velocity and acceleration?

$$v = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

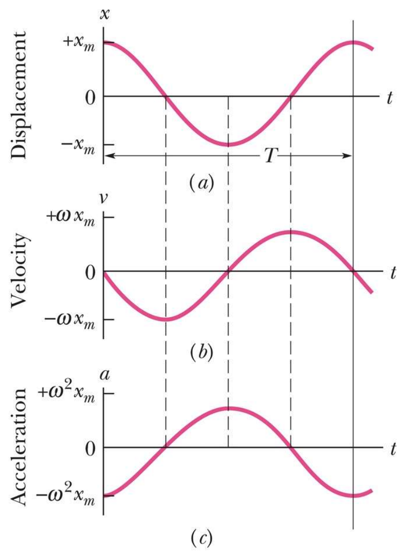
$$v(t) = -\omega x_m \sin(\omega t)$$

$$a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a(t) = -\omega^2 x_m \cos(\omega t)$$

$$a(t) = -\omega^2 x(t)$$

The signs matter…



Extreme values here mean …

zero values here and …

extreme values here.

Example 1:

- In an electric shaver, the blade moves back and forth over a distance of *2.0 mm* in a simple harmonic motion with a frequency of *120 Hz*. Find,

  a) the amplitude of the motion, and

  b) the maximum blade speed, and

  c) the magnitude of the maximum blade acceleration.

Example 1: Work the Example

What does this have to do with springs?

- Restoring forces fights the acceleration.

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x$$

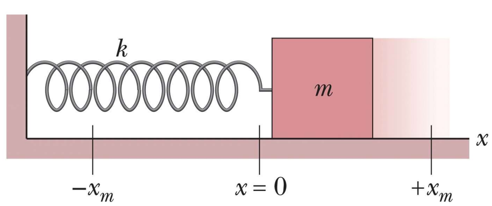
- Relate this to Hooke's Law from last quarter:

$$F = -kx \quad \text{and so, } k = m\omega^2$$

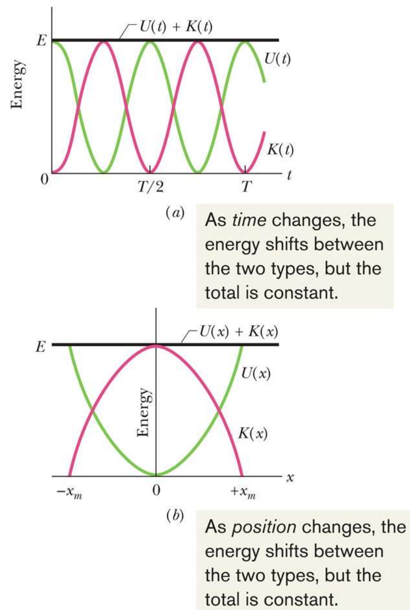
- And:

$$\omega = \sqrt{(k/m)}$$

$$T = 2\pi \sqrt{(m/k)}$$



Energy in Simple Harmonic Motion



Energy in Simple Harmonic Motion

- Total energy always remains the same:

$$E = U + K = \tfrac{1}{2}kx_m^2.$$

- But since:

$$x(t) = x_m \cos(\omega t + \phi)$$

- U and K can have different values when we look at them as functions of time:

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

Example 2:

- Find the mechanical energy of a block-spring system with a spring constant of $1.3\ N/cm$ and an amplitude of $2.4\ cm$.

Example 2: Work the Example